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Simulation of Multi-Platform Geolocation using a Hybrid TDOA/AOA Method

Huai-Jing Du and Jim P.Y. Lee

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Abstract

The problem of geolocation of radar emitters has been studied extensively in the past decades. Several location methods such as the methods based on measuring frequency difference of arrival (FDOA), time difference of arrival (TDOA), angle of arrival (AOA), etc., have been investigated in the past. However, each method has its own advantages and limitations in terms of location accuracy. This report presents a multi-platform geolocation scheme by combining TDOA and AOA methods for three-dimensional (3-D) geolocation of radar emitters. A mathematical model is developed by combining sensor information such as AOA measurements, TDOA measurements and sensor position information from all platforms. A least-squares (LS) solution based on the model is derived by processing the sensor information. Through the use of the proposed hybrid TDOA/AOA location scheme, location ambiguity due to using TDOA method alone can be resolved and better location accuracy can be achieved. In addition, unlike single platform geolocation techniques, multi-platform geolocation generally results in quick localization of radar emitters. Therefore, it will reduce response time to incoming threats and improve situation awareness. A Monte-Carlo simulation is presented using additive white Gaussian noise to demonstrate the performance of the proposed hybrid TDOA/AOA geolocation method.

Résumé

Le problème de géolocation d'émetteurs radars a été étudié de façon extensive au cours des décennies précédentes. Plusieurs techniques de localisation, telles que la mesure de la différence de fréquence à l'arrivée (FDOA), de la différence de temps à l'arrivée (TDOA) et de l'angle d'arrivée (AOA), ont été investiguées par le passé. Cependant, chaque méthode a ses avantages propres, ainsi que ses limites en termes de précision en ce qui regarde la localisation. Ce rapport présente une approche multi plate-formes en combinant les méthodes TDOA et AOA pour la localisation 3-D d'émetteurs radar. Un modèle mathématique est présenté, combinant pour toutes les plate-formes, l'information provenant des détecteurs telle que les mesures de AOA, TDOA ainsi que leur position. Une solution par moindres carrés (LS) basée sur le modèle est dérivée en traitant l'information en provenance des détecteurs. En utilisant la combinaison hybride TDOA/AOA proposée, l'ambiguïté de localisation causée par TDOA seulement peut être résolue et une plus grande précision dans la localisation spatiale peut être atteinte. Contrairement à la localisation basée sur une seule plate-forme, la localisation multi plate-formes fournira des mesures rapides et précises et, par conséquent, améliorera la connaissance du terrain ainsi que le temps de réponse aux menaces. Une simulation de Monte-Carlo est présentée pour démontrer la performance de la technique de localisation hybride proposée. L'ajout de bruit blanc gaussien est utilisé durant cette simulation.

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Executive summary

The requirement for passive geolocation of radar emitters remains an important problem in electronic support (ES) for maritime and littoral surveillance. Several location techniques such as the ones based on measuring frequency difference of arrival (FDOA), time difference of arrival (TDOA), angle of arrival (AOA), etc., have been studied extensively in the past. However, in general, there is no single method that will provide accurate location estimation under all circumstances. Each method has its own advantages and limitations in terms of location accuracy.

Both TDOA and FDOA location methods generally provide better location accuracy than the AOA-based location method [2]. However, the TDOA-based method requires relatively more sensors than the AOA-based method to perform geolocation of radar emitters. For instance, the TDOA-based method requires at least three properly distributed sensors for two-dimensional (2-D) geolocation. In addition, it requires that the TDOA measurements at all sensors be performed on the same pulse from the same pulse train. Otherwise, location ambiguity will be introduced if the TDOA measurements are taken on different transmitted pulses. The FDOA-based method requires relative movement between the transmitter and the receiver. The AOA method, on the other hand, requires two sensors for 2-D localization. However, it is highly range dependent and a small error in the angle measurements will result in a large location error if the sensor is far away from the transmitter. Therefore, to achieve better location accuracy, a combination of two or more location schemes should be considered in order to complement each other.

This report presents a multi-platform geolocation scheme by combining TDOA and AOA location methods for three-dimensional (3-D) geolocation of radar emitters. The proposed hybrid TDOA/AOA location scheme is achieved by utilizing spatially separated sensors on different platforms, such as UAVs and ship/land-based platforms, to improve detection range and location accuracy. The radar emitter geolocation is performed by processing the information available from each sensor, including AOA measurements, TDOA measurements and sensor position information. Combining the AOA method with the TDOA method can assist in eliminating location ambiguity associated with TDOA alone and can enhance location accuracy. In addition, the proposed location scheme, unlike single platform utilizing individual location method, will result in quick and better location estimates of radar emitters. Therefore, it will improve ES operation and situation awareness.

The theoretical performance and effectiveness of the proposed location scheme is demonstrated using Monte-Carlo simulation with additive white Gaussian noise. However, this study did not take into consideration of some of the real-life problems caused by multipath, time of arrival ambiguity between sensors, asynchronous illumination of the sensors by an emitter and UAV trajectory selection. Some of these issues will be addressed in the future.

Du, H.J. and Lee, J. P.Y. 2004. Simulation of Multi-Platform Geolocation using a Hybrid TDOA/AOA Method. DRDC Ottawa TM 2004-256. Defence R&D Canada - Ottawa.

Sommaire

L'exigence de la géolocalisation passive d'émetteurs radar demeure un problème important dans le domaine du soutien électronique (SE) de la surveillance maritime et côtière. Par le passé, plusieurs techniques de localisation, telles que celles basées sur la mesure de la différence de fréquence à l'arrivée (FDOA), de la différence de temps à l'arrivée (TDOA) et de l'angle d'arrivée (AOA), ont été étudiées de façon exhaustive. Toutefois, en règle générale, il n'y a pas de méthode unique qui donne une estimation précise de la position. Chaque méthode comporte ses propres avantages et limitations en termes de précision de la localisation en toutes circonstances. Par exemple, la méthode TDOA exige en général au moins trois capteurs répartis de façon appropriée pour la localisation bidimensionnelle (2D) avec ambiguïté possible de la localisation; la méthode FDOA exige le mouvement relatif entre un émetteur et un récepteur. Les méthodes TDOA et FDOA ont généralement une meilleure précision que le système AOA. Par contre, le système AOA n'exige que deux capteurs pour l'estimation de la position en 2D. Cependant, il est extrêmement sensible à la distance, et une faible erreur des mesures d'angle donne lieu à une grande erreur de localisation si le capteur se trouve à une grande distance d'un émetteur. Pour augmenter la précision de la localisation, on devrait envisager une combinaison de deux ou plusieurs systèmes de localisation pour obtenir l'avantage de différentes techniques de localisation complémentaires.

Dans le présent rapport, nous proposons un système de localisation au moyen de la combinaison des méthodes TDOA et AOA pour la géolocalisation tridimensionnelle (3D) d'émetteurs radar. Le système TDOA/AOA hybride proposé est réalisé grâce à l'utilisation de capteurs séparés dans l'espace (long axe de référence) sur différentes plates-formes pour augmenter la portée de détection/couverture et améliorer la précision des mesures. Nous présentons un modèle mathématique en combinant l'information des capteurs, telle que les mesures AOA, les mesures TDOA et l'information de position des capteurs de toutes les plates-formes. Pour localiser les émetteurs radar, une solution des moindres carrés (LS) basée sur le modèle est dérivée par le traitement de l'information des capteurs. Grâce à la combinaison des méthodes TDOA et AOA, on peut éliminer l'ambiguïté de la localisation due à la TDOA utilisée seule et obtenir une meilleure précision de la localisation que celle de l'AOA utilisée seule. En outre, au contraire de la localisation à plate-forme unique, la localisation à plusieurs plates-formes produira des mesures rapides et précises et améliorera donc la conscience de la situation et le temps de réponse aux menaces. Nous présentons une simulation Monte Carlo pour faire la démonstration de la technique proposée tout en utilisant du bruit blanc gaussien additif.

La performance théorique du plan proposé de localisation est obtenue au moyen d'une simulation informatique qui prévoit un biais et un bruit additifs. Il est confirmé que des estimations plus précises de l'emplacement sont obtenues parce que le plan proposé de localisation permet une estimation et le retrait subséquent du biais des mesures. Toutefois, la présente étude n'a pas tenu compte de certains des problèmes réels dus à l'ambiguïté causée par la propagation par trajets multiples ou les écarts de temps

d'arrivée aux capteurs, à l'illumination asynchrone des capteurs par un émetteur ou à la sélection de la trajectoire des VAT.

Du, H. J. and Lee, J. P. Y. 2004. Simulation of Multi-Platform Geolocation using a Hybrid TDOA/AOA Method. DRDC Ottawa TM 2004-256. R & D pour la défense Canada - Ottawa.

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1. Introduction

The requirement for passive geolocation of radar emitters remains an important problem in electronic support (ES) for maritime and littoral surveillance. Several location techniques have been extensively studied and investigated in the past including angle of arrival (AOA) [1], time difference of arrival (TDOA) [2][3] and frequency difference of arrival (FDOA) [4][5][6]. In general, there is no single geolocation method that will give accurate location estimation under all circumstances. Each location technique has its own advantages and disadvantages. The TDOA method is based on measuring the time difference of arrival of incoming signals at two or more sensors. The TDOA method, in general, requires at least three properly distributed sensors for two-dimensional (2-D) location but it may cause location ambiguity if the TDOA measurements are performed on different pulses of a pulse train. The FDOA method is based on measuring the frequency difference of received signals at two or more sensors and it requires relative movement between sensors and a transmitter. In general, both TDOA and FDOA have better location accuracy than the AOA method [7]. The AOA method, on the other hand, requires only two sensors minimum for a 2-D location estimate. However, it is highly range dependent. A small error in the angle measurements will result in a large location error if the sensor is far away from a transmitter. To achieve better location accuracy, a combination of two or more location methods should be considered in order to complement each other.

Fang [8] and Schau *et al.* [9] derived a closed-form location solution when the number of TDOA measurements is equal to the number of unknowns (i.e., the coordinates of a transmitter). This solution, however, cannot make use of extra measurements when they become available to improve position accuracy. The more general situation with extra measurements by utilizing multiple stationary sensors was considered by Smith *et al.* [10]. To obtain a precise position estimate with reasonable noise levels, the Taylor-series method [3][12] is commonly employed. Du *et al.* [13] proposed a Taylor-series based location method by utilizing one or two moving sensors. The proposed method takes into account both random noise and measurement bias, but it requires an initial guess of the starting point to start iterative location estimation. Chan *et al.* [11] proposed an explicit approach based on the two-stage least-squares computation for the case when the TDOA estimation error is small. To incorporate AOAs for a better position estimate, Cong *et al.* [7] proposed a hybrid TDOA/AOA scheme for 2-D mobile user localization for a wireless communication application. Their solution is based on a two-stage least-squares algorithm [11] for the case when the TDOA and AOA measurement errors are small. In this report, we propose a hybrid TDOA/AOA location scheme which extends the approach in [7] for a 2-D location problem to 3-D geolocation of radar emitters.

The proposed hybrid TDOA/AOA scheme is achieved through combining the TDOA location with AOA location. As the TDOA and AOA location methods complement each other, the proposed location scheme has the advantages of both TDOA and AOA methods to achieve better location accuracy. The proposed location scheme utilizes spatially separated sensors (long baseline) on different platforms to increase detection

range/coverage and enhance measurement accuracy. A mathematical model is presented by combining sensor information, such as AOAs, TDOAs and sensor position information from all platforms. A least-squares (LS) location solution based on the model is derived by processing the information available from each sensor. By the combination of TDOA and AOA methods, the location ambiguity associated with the TDOA method alone can be resolved and better location accuracy can be achieved. In addition, the proposed multi-platform location scheme, unlike single platform using individual location method, will result in quick location of radar emitters. Therefore, it will reduce response time to incoming threats and improve situation awareness. The theoretical performance of the proposed location scheme is demonstrated using Monte-Carlo simulation with additive white Gaussian noise.

2. Location Methods – TDOA, AOA and Hybrid TDOA/AOA

2.1 The TDOA Method

Fig.1 illustrates the geometry of TDOA technique using multiple stationary sensors. The time of arrival (TOA) is defined as the time interval for signals propagating from the transmitter to a sensor, as shown in Fig.2. The TOA is dependent on transmitter-sensor geometry and medium characteristics. In the case of a constant velocity medium as assumed in this study, the time interval is a function of the distance/range between the transmitter and the sensor [10]. Let t_i be the time interval for the signal radiating from the emitter to arrive at sensor i and r_i be the distance/range between the emitter and the sensor i, i.e.,

$$r_i = \sqrt{(x_e - x_i)^2 + (y_e - y_i)^2 + (z_e - z_i)^2}\ . \tag{1}$$

The TDOA between sensors i and j is then defined by $\Delta t_{i,j}$ as

$$\Delta t_{i,j} = t_i - t_j = \frac{1}{c}(r_i - r_j), \tag{2}$$

where c is the speed of light, (x_i, y_i, z_i) is the position coordinate of sensor i and (x_e, y_e, z_e) is the position coordinate of the emitter. For simplicity, we assume that the TDOAs are obtained with respect to the first sensor, i.e., the index j is presumed to be 1 and the index i is presumed to run from 2 to k. In Eq.2, the $\Delta t_{i,j}$ is considered as the error-free values of TDOAs. In practice, however, noise is always present in the TDOA measurements. Therefore, Eq.2 can be rewritten as

$$\Delta t_{i,1} = \frac{r_i - r_1}{c} + n_{\Delta t_{i,1}}, \qquad i = 2,3,\ldots,k, \tag{3}$$

where $n_{\Delta t_{i,1}}$ is the measurement noise associated with $\Delta t_{i,1}$. In order to solve three unknowns (x_e, y_e, z_e), a minimum of three TDOA measurements from four stationary sensors is required.

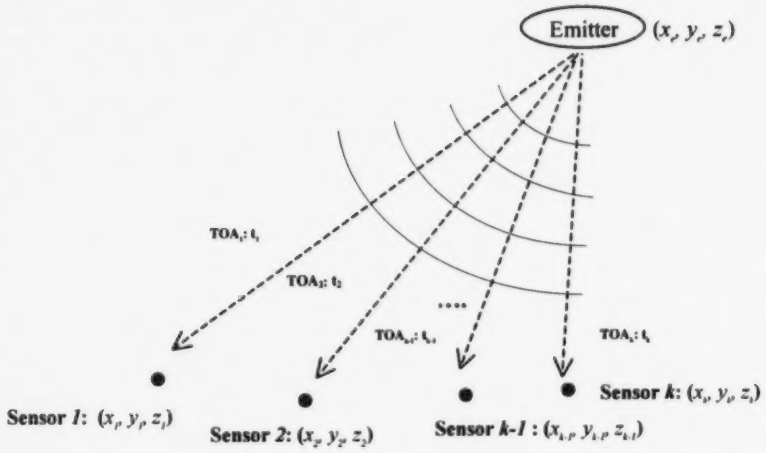


Figure 1. *The geometry of TDOA technique using multiple stationary sensors*

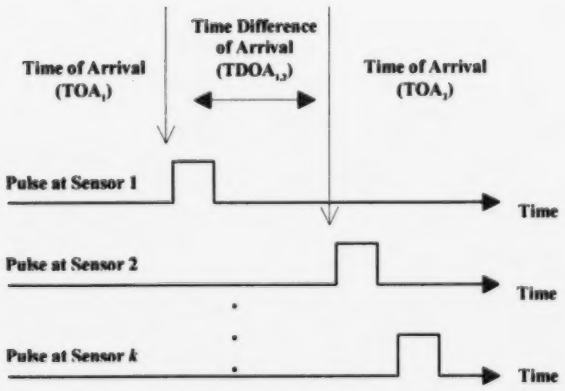


Figure 2. *Time of arrivals (TOA) and time differences of arrival (TDOA)*

2.2 The AOA Method

Fig. 3 shows the geometry of the AOA location technique in the Cartesian coordinate. The AOA method is based on measuring the angles of arrivals of the received signals in terms of azimuth angle θ and elevation angle ϕ from the sensor to the emitter. The nonlinear relationship between the AOAs and the sensor/emitter coordinates are expressed by

$$\begin{aligned} \theta_0 &= \tan^{-1}\left(\frac{y_e - y_s}{x_e - x_s}\right), \\ \phi_0 &= \tan^{-1}\left(\frac{z_e - z_s}{\sqrt{(x_e - x_s)^2 + (y_e - y_s)^2}}\right), \end{aligned} \tag{4}$$

where θ_0 and ϕ_0 are defined as the error-free values of the azimuth and elevation angles, (x_s, y_s, z_s) are the sensor position coordinates, and (x_e, y_e, z_e) are the emitter location coordinates. In practice, however, noise is always present in the AOA measurements. Therefore, Eq.4 can be rewritten as

$$\begin{aligned} \theta &= \theta_0 + n_\theta, \\ \phi &= \phi_0 + n_\phi, \end{aligned} \tag{5}$$

or

$$\begin{aligned} n_\theta &= \theta - \theta_0, \\ n_\phi &= \phi - \phi_0, \end{aligned} \tag{6}$$

where θ and ϕ are the azimuth and elevation measurements, and n_θ and n are the noise related to the azimuth and elevation angle measurements. Taking the sine of both sides of Eq.6 and multiplying by l_1 and l_2 respectively, we have

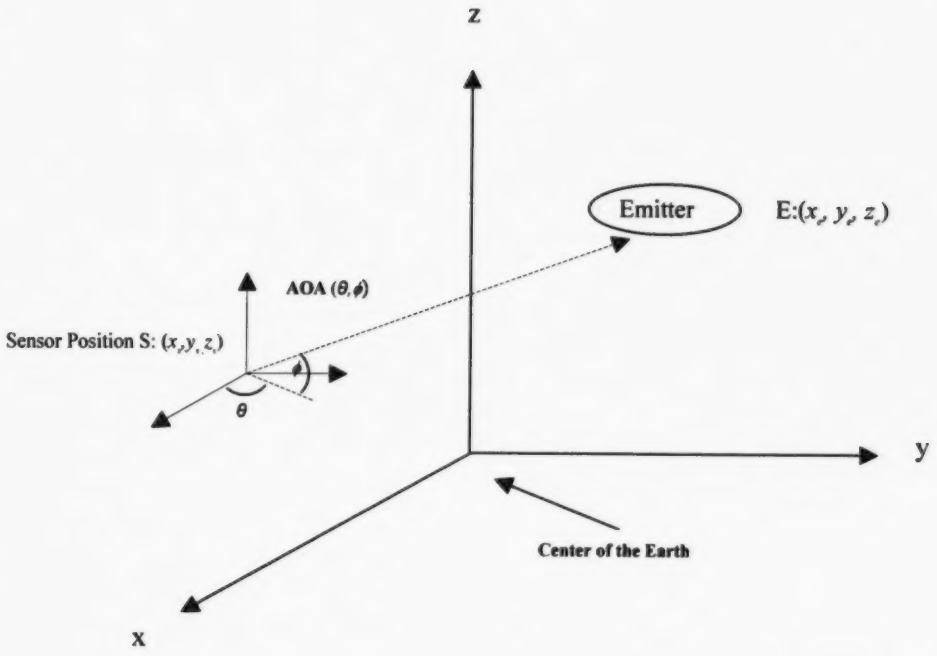


Figure 3. *The geometry of AOA location technique in terms of the azimuth angle θ and the elevation angle ϕ of received signals at the sensor*

$$l_1 \sin n_\theta = l_1 \sin(\theta - \theta_0),$$
$$l_2 \sin n_\phi = l_2 \sin(\phi - \phi_0), \tag{7}$$

where $l_1 = \sqrt{(x_e - x_s)^2 + (y_e - y_s)^2}$ is defined as the length of x-y plane projection of vector $\overline{SE}$ and $l_2 = \sqrt{(x_e - x_s)^2 + (y_e - y_s)^2 + (z_e - z_s)^2}$ is the distance between the emitter and the sensor in the Cartesian coordinate given in Fig. 3. Considering Fig.3, we have

$$
\begin{aligned}
x_e - x_s &= l_1 \cos\theta_0, \\
y_e - y_s &= l_1 \sin\theta_0,
\end{aligned}
\tag{8}
$$

and

$$
\begin{aligned}
x_e - x_s &= l_2 \cos\phi_0 \cos\theta_0, \\
y_e - y_s &= l_2 \cos\phi_0 \sin\theta_0, \\
z_e - z_s &= l_2 \sin\phi_0.
\end{aligned}
\tag{9}
$$

Expanding Eq.7 and then substituting Eqs. 8 and 9 into Eq.7 gives

$$
\begin{aligned}
l_1 \sin n_\theta &= (x_e - x_s)\sin\theta - (y_e - y_s)\cos\theta, \\
l_2 \sin n_\phi &= (x_e - x_s)\frac{\sin\phi}{\cos\theta_0} - (z_e - z_s)\cos\phi.
\end{aligned}
\tag{10}
$$

To simplify the formulation of the problem, we assume that AOA measurement noise are very small, i.e., $|n_\theta|$ and $|n\ \ |<<1$ and therefore, $\sin(n_\theta) \approx n_\theta$, $\sin(n\ \ ) \approx n$ and $\cos(\theta_0) \approx \cos(\theta)$. Based on these assumptions, we can approximately rewrite Eq.10 in a linear form as

$$
\begin{aligned}
0 &\approx -(x_e - x_s)\sin\theta + (y_e - y_s)\cos\theta + l_1\ n_\theta, \\
0 &\approx -(x_e - x_s)\frac{\sin\phi}{\cos\theta} + (z_e - z_s)\cos\phi + l_2\ n_\phi.
\end{aligned}
\tag{11}
$$

or

$$
\begin{aligned}
0 &\approx -\frac{(x_e - x_s)\sin\theta}{l_1} + \frac{(y_e - y_s)\cos\theta}{l_1} + n_\theta, \\
0 &\approx -\frac{(x_e - x_s)\sin\phi}{l_2\cos\theta} + \frac{(z_e - z_s)\cos\phi}{l_2} + n_\phi.
\end{aligned}
\tag{12}
$$

As can be seen from Fig.3, Eq.12 is derived based on the angle measurements from a single sensor. In the case of multiple sensors, e.g., k sensors, Eq.12 can be rewritten as a set of equations as

$$
\begin{aligned}
0 &\approx -\frac{(x_e - x_{n,s})\sin\theta_n}{l_{n,1}} + \frac{(y_e - y_{n,s})\cos\theta_n}{l_{n,1}} + n_{\theta_n}, \\
0 &\approx -\frac{(x_e - x_{n,s})\sin\phi_n}{l_{n,2}\cos\theta} + \frac{(z_e - z_{n,s})\cos\phi_n}{l_{n,2}} + n_{\phi_n}, \\
& n = 1,\ldots,k,
\end{aligned} \tag{13}
$$

where the subscript n in Eq.13 denotes the n^{th} sensor.

2.3 The Hybrid TDOA/AOA Method

Combining Eq.3 and Eq.13, and then re-arranging it in a matrix form with $i = 2,\ldots,k$ and $n = 1,2,\ldots,k$, we have

$$
\boldsymbol{m} = \boldsymbol{g}(\boldsymbol{X}) + \boldsymbol{n} \ , \tag{14}
$$

where $\boldsymbol{X} = [x_e \ y_e \ z_e]^T$ is the vector of the unknown variables and

$$
\boldsymbol{m} = \begin{bmatrix} \Delta t_{2,1} \\ \Delta t_{3,1} \\ \vdots \\ \Delta t_{k,1} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \boldsymbol{g}(\boldsymbol{X}) = \begin{bmatrix} (r_2 - r_1)/c \\ (r_3 - r_1)/c \\ \vdots \\ (r_k - r_1)/c \\ -(x_e - x_{1,s})\sin\theta_1 / l_{1,1} + (y_e - y_{1,s})\cos\theta_1 / l_{1,1} \\ -(x_e - x_{1,s})\sin\phi_1 /(l_{1,2}\cos\theta_1) + (z_e - z_{1,s})\cos\phi_1 / l_{1,2} \\ \vdots \\ -(x_e - x_{k,s})\sin\theta_k / l_{k,1} + (y_e - y_{k,s})\cos\theta_k / l_{k,1} \\ -(x_e - x_{k,s})\sin\phi_k /(l_{k,2}\cos\theta_k) + (z_e - z_{k,s})\cos\phi_k / l_{k,2} \end{bmatrix}, \tag{15}
$$

and $\boldsymbol{n}$ is the vector of TDOA and AOA measurement noises, i.e.,

$$\boldsymbol{n} = [n_{\Delta t_{2,1}} \cdots n_{\Delta t_{k,1}} \; n_{\theta_1} \, n_{\phi_1} \cdots n_{\theta_k} \, n_{\phi_k}]^{\mathrm{T}} \quad . \tag{16}$$

In the following, an LS solution based on the combination of TDOA and AOA measurements is introduced to solve this nonlinear problem.

3. Multi-Platform Geolocation Using the Hybrid TDOA/AOA Method

3.1 Geolocation from UAV and Ship/Land-Based Sensors

Unmanned Aerial Vehicles (UAVs) today are gaining wide acceptance as valuable reconnaissance, surveillance, targeting and intelligence gathering tools to support various missions. However, one of the common uses for a UAV today is to find a target and determine its location. Fig. 4 illustrates a possible operational scenario of passive geolocation from UAV and ship/land-based sensors using the proposed hybrid TDOA/AOA method.

3.2 Formulation of the Problem

Instead of using multiple stationary sensors, Fig.4 shows the geolocation scenario where two time-synchronized ESM sensors are utilized with sensor 1 on the ship/land-based platform and sensor 2 on the UAV platform. The multiple TDOA measurements are collected from the two sensors over a time period along the UAV path. The multiple AOA measurements (i.e., elevation and azimuth angles) are obtained from the ship/land-based sensor.

For simplicity of analyzing the problem, Fig.4 is re-illustrated in Fig.5 with sensor 1 being the origin (i.e., $x = 0$, $y = 0$ and $z = 0$) of the coordinate. The coordinates of sensor 2 are defined by $(x_{i,2}, y_{i,2}, z_{i,2})$ at the i^{th} time instant along the UAV trajectory.

- ***Multiple TDOA Equations***

Eq.14 defines a set of nonlinear equations as a function of unknown variables (x_e, y_e, z_e). Since solving these nonlinear equations is difficult, an alternative [11] is to transform these equations into another set of linear equations through introducing another unknown variable, r_1.

For simplicity of the following derivation, re-writing Eq.3 without consideration of TDOA measurement noise gives

$$\Delta t_{i,1} = \frac{r_i - r_1}{c}, \qquad i = 2,3,\ldots,k, \tag{17}$$

or

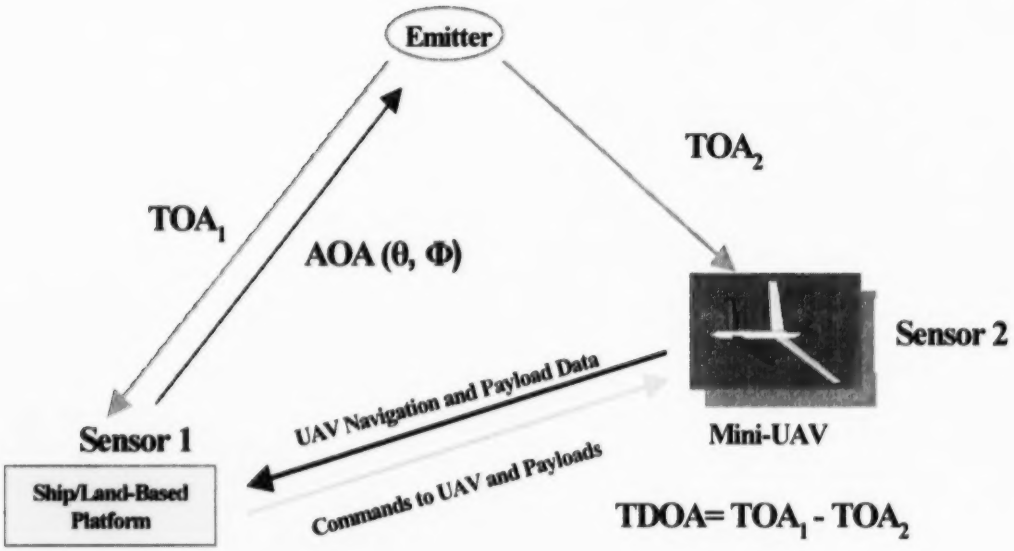


***Figure 4**. Geolocation using the hybrid TDOA/AOA method from a mini-UAV in conjunction with a ship/land-based platform*

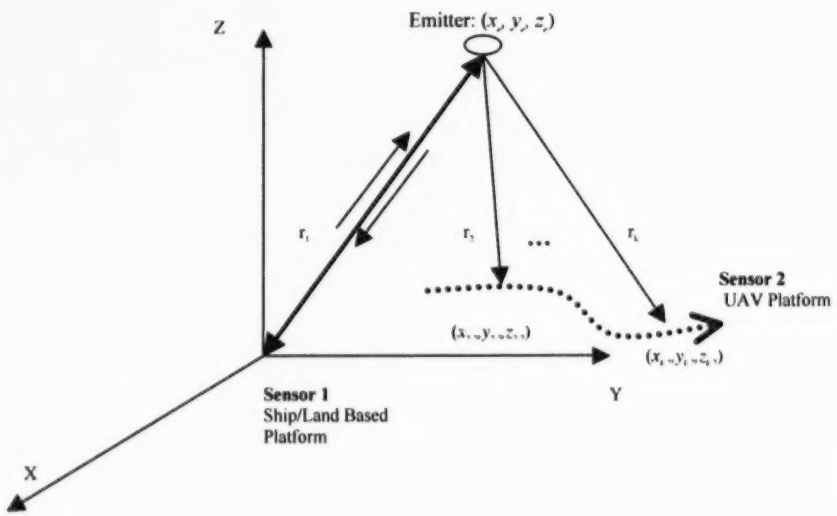


***Figure 5.** Geometry between an emitter and two sensors.*

$$c\Delta t_{i,1} = r_i - r_1, \quad i = 2,3,...,k, \tag{18}$$

where r_1 is defined as the range between sensor 1 and the emitter, and r_i is the range between sensor 2 and the emitter at the i^{th} time instant where $i \neq 1$ (i = 2,..., k) to distinguish r_i from r_1.

In the following, we define $r_{i,1}$ as the range difference between r_i and r_1 at the i^{th} time instant along the UAV path. Therefore, Eq.18 can be rewritten as

$$r_{i,1} = c\Delta t_{i,1} = r_i - r_1, \quad i = 2,3,...,k. \tag{19}$$

Re-arranging Eq.19 and then taking the square of both sides of the equation gives

$$r_i^2 = (r_{i,1} + r_1)^2, \quad i = 2,3,...,k. \tag{20}$$

As can be seen from Fig.5, we have

$$r_1^2 = x_e^2 + y_e^2 + z_e^2 , \tag{21}$$

and

$$r_i^2 = (x_{i,2} - x_e)^2 + (y_{i,2} - y_e)^2 + (z_{i,2} - z_e)^2, \quad i = 2,3,...,k. \tag{22}$$

where $(x_{i,2}, y_{i,2}\ z_{i,2})$ are the coordinates of sensor 2 at the i^{th} time instant along the UAV trajectory. Substituting Eqs.21 and 22 into Eq.20 gives

$$\frac{1}{2}({r_{i,1}}^2 - K_i) = -x_{i,2}x_e - y_{i,2}y_e - z_{i,2}z_e - r_{i,1}r_1, \qquad i = 2,3,...k, \tag{23}$$

where

$$K_i = x_{i,2}^2 + y_{i,2}^2 + z_{i,2}^2 . \tag{24}$$

Rewriting Eq.23 with $i = 2,3,\ldots,k$ in a matrix form gives

$$\boldsymbol{m}_1 = \boldsymbol{g}_1 \boldsymbol{X}, \tag{25}$$

where

$$\boldsymbol{m}_1 = \frac{1}{2}\begin{bmatrix} {r_{2,1}}^2 - K_2 \\ {r_{3,1}}^2 - K_3 \\ \vdots \\ {r_{k,1}}^2 - K_k \end{bmatrix}, \qquad \boldsymbol{g}_1 = \begin{bmatrix} -x_{2,2} & -y_{2,2} & -z_{2,2} & -r_{2,1} \\ -x_{3,2} & -y_{3,2} & -z_{3,2} & -r_{3,1} \\ \vdots & \vdots & \vdots & \vdots \\ -x_{k,2} & -y_{k,2} & -z_{k,2} & -r_{k,1} \end{bmatrix}, \tag{26}$$

and $\boldsymbol{X} = [x_e \ \ y_e \ \ z_e \ r_1]^T$ is the vector of unknown variables x_e, y_e, z_e and r_1.

As can be seen, Eq. 25 is derived without consideration of TDOA noise. In practice, however, noise is always present in the measurements. To have better formulation of the problem, an error vector $\boldsymbol{\varphi}_1$ is introduced into Eq.25 as

$$\boldsymbol{m}_1 = \boldsymbol{g}_1 \boldsymbol{X} + \boldsymbol{\varphi}_1 . \tag{27}$$

- ***Multiple AOA Equations***

Considering Eq.13 with sensor 1 at the coordinate origin as shown in Fig.5, i.e., ($x_{1,s}$, $y_{1,s}$, $z_{1,s}$)= (*0*, *0*, *0*), we have AOA equations as

$$\begin{aligned} 0 &= -x_e \sin\theta_i + y_e \cos\theta_i + l_{i,1}\, n_{\theta_i}, \\ 0 &= -x_e \frac{\sin\phi_i}{\cos\theta_i} + z_e \cos\phi_i + l_{i,2}\, n_{\phi_i}, \end{aligned} \tag{28}$$

where θ_i and ϕ_i are the azimuth and elevation angles measured at sensor 1 at the i^{th} time instant. Eq.28 can be rewritten in a matrix form with $i = 2,3,\ldots,k$ as

$$\boldsymbol{m}_2 = \boldsymbol{g}_2 \boldsymbol{X} + \boldsymbol{\varphi}_2, \tag{29}$$

where

$$\boldsymbol{X} = [x_e \;\; y_e \;\; z_e \;\; r_1]^{\mathrm{T}}, \tag{30}$$

is the vector of unknown variables and

$$\boldsymbol{\varphi}_2 = [l_{2,1} n_{\theta_2} \;\; l_{2,2} n_{\phi_2} \;\; \cdots l_{k,1} n_{\theta_k} \;\; l_{k,2} n_{\phi_k}]^{\mathrm{T}}, \tag{31}$$

is the error vector associated with the AOA equations, and

$$\boldsymbol{m}_2 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \boldsymbol{g}_2 = \begin{bmatrix} -\sin\theta_2 & \cos\theta_2 & 0 & 0 \\ -\sin\phi_2/\cos\theta_2 & 0 & \cos\phi_2 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ -\sin\theta_k & \cos\theta_k & 0 & 0 \\ -\sin\phi_k/\cos\theta_k & 0 & \cos\phi_k & 0 \end{bmatrix}. \tag{32}$$

- ***Multiple Hybrid TDOA/AOA Equations***

Combining Eq.25 and Eq.29 gives

$$\begin{bmatrix} \boldsymbol{m}_1 \\ \boldsymbol{m}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{g}_1 \\ \boldsymbol{g}_2 \end{bmatrix} \boldsymbol{X} + \begin{bmatrix} \boldsymbol{\varphi}_1 \\ \boldsymbol{\varphi}_2 \end{bmatrix}. \tag{33}$$

Eq.33 represents a set of overdetermined location equations as the number of measurements is greater than the number of unknowns. In the presence of noise, this set of equations will not meet at the same point and the proper solution is the (x_e, y_e, z_e, r_1) that best fit these equations.

3.3 The Least-Squares Approach

The vector $\boldsymbol{X}$ of unknown variables x_e,y_e, z_e and r_1 can then be estimated by the following weighted LS calculation

$$\hat{\boldsymbol{X}} = \arg\min_{\boldsymbol{X}} \left\| \boldsymbol{W}^{1/2} \left(\begin{bmatrix} \boldsymbol{m}_1 \\ \boldsymbol{m}_2 \end{bmatrix} - \begin{bmatrix} \boldsymbol{g}_1 \\ \boldsymbol{g}_2 \end{bmatrix} \boldsymbol{X} \right) \right\|_2^2, \tag{34}$$

i.e.,

$$\hat{\boldsymbol{X}} = \left(\begin{bmatrix} \boldsymbol{g}_1 \\ \boldsymbol{g}_2 \end{bmatrix}^T \boldsymbol{W} \begin{bmatrix} \boldsymbol{g}_1 \\ \boldsymbol{g}_2 \end{bmatrix} \right)^{-1} \begin{bmatrix} \boldsymbol{g}_1 \\ \boldsymbol{g}_2 \end{bmatrix}^T \boldsymbol{W} \begin{bmatrix} \boldsymbol{m}_1 \\ \boldsymbol{m}_2 \end{bmatrix}. \tag{35}$$

The emitter position coordinates x_e, y_e and z_e can be picked out by pre-multiplying the both sides of Eq.35 by a 3×4 matrix composed mostly of zeroes as

$$\begin{bmatrix} \hat{x}_e \\ \hat{y}_e \\ \hat{z}_e \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \left(\begin{bmatrix} \boldsymbol{g}_1 \\ \boldsymbol{g}_2 \end{bmatrix}^T W^{-1} \begin{bmatrix} \boldsymbol{g}_1 \\ \boldsymbol{g}_2 \end{bmatrix} \right)^{-1} \begin{bmatrix} \boldsymbol{g}_1 \\ \boldsymbol{g}_2 \end{bmatrix}^T W^{-1} \begin{bmatrix} \boldsymbol{m}_1 \\ \boldsymbol{m}_2 \end{bmatrix}. \tag{36}$$

Note $\boldsymbol{W}$ in Eq.36 is defined as a weighting matrix. In the case where $\boldsymbol{\varphi}_1$ and $\boldsymbol{\varphi}_2$ in Eq.33 are Gaussian random vectors and $\boldsymbol{W}$ is suitably chosen as the covariance matrix of $[\boldsymbol{\varphi}_1{}^T \boldsymbol{\varphi}_2{}^T]^T$, i.e., $\boldsymbol{W} = \mathbf{E}([\boldsymbol{\varphi}_1{}^T \boldsymbol{\varphi}_2{}^T]^T[\boldsymbol{\varphi}_1{}^T \boldsymbol{\varphi}_2{}^T])$, then the position estimation in Eq.36 becomes a maximum-likelihood (ML) estimate [11]. However, the weighting matrix required in LS is not easy to determine and therefore it is not discussed in this report. To simplify the problem, we define the weighing matrix $\boldsymbol{W} = \mathbf{I}$ (an identity matrix) here.

The LS approach to solve the nonlinear problem is based on the assumption of independence among x_e,y_e, z_e and r_1. They are then solved by the LS given in Eq.36. An improved estimate of the unknown variables can be obtained via another LS computation [11] by applying the computed result $(\hat{x}_e, \hat{y}_e, \hat{z}_e)$ to the known relationship in Eq.21 to solve r_1.

4. Monte-Carlo Simulation

The performance of the proposed location scheme has been evaluated using computer simulation. One type of computer simulation is Monte-Carlo simulation. Monte-Carlo simulation is a stochastic technique used to investigate and solve mathematical problems where uncertain or random variables are presented. The term "Monte Carlo" comes from the name of a city in Monaco [14] where main attractions are casinos running games by exploiting the random behaviour of each game. Monte-Carlo methods were originally developed for the Manhattan Project during World War II because of the similarity of statistical simulation to games of chance. However, they are now applied to a wide range of problems.

In the following, Monte-Carlo simulation was used to evaluate the performance of the proposed geolocation scheme with the consideration of TDOA and AOA measurement noise. The measurement noise for TDOAs and AOAs were assumed to be white noise with Gaussian distribution. During each run of the Monte-Carlo simulation, random measurement noise was generated based on its defined statistical distribution (such as normal distribution, mean and variance) and then emitter position was estimated based on this randomly generated noise.

In the following, the simulation was performed based on the geolocation scenarios given in Figs.4 and 5. For each simulation scenario, 10,000 simulation runs were performed and the emitter location estimates $(\hat{x}_e, \hat{y}_e, \hat{z}_e)$ were then computed by averaging the 10,000 location estimate results over the 10,000 simulation runs.

To illustrate the accuracy of location estimation, the location estimation error is used and defined by

$$\text{Location Estimation Error} = \sqrt{(\hat{x}_e - x_e)^2 + (\hat{y}_e - y_e)^2 + (\hat{z}_e - z_e)^2}. \qquad (37)$$

where $(\hat{x}_e, \hat{y}_e, \hat{z}_e)$ are the location estimates of the emitter and (x_e, y_e, z_e) are the emitter true positions.

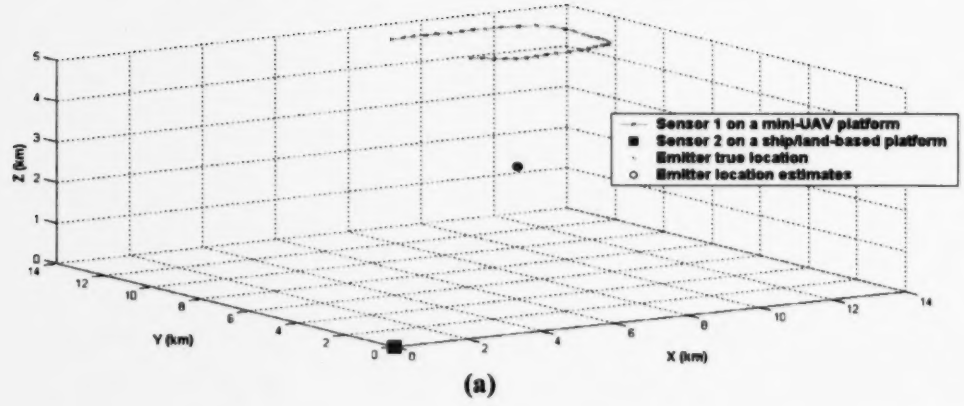
Simulation scenario: geolocation from a ship/land-based sensor accompanied by a mini-UAV based sensor

In the following, Figs. 6(a) and 7(a) show the geometry between the emitter and the sensors as well as the UAV sensor trajectory. We assume that the emitter's true position is at (x_e, y_e, z_e)=(10.0, 10.0, 2.0) kilometers (km) in the coordinate given in Figs. 6(a) and 7(a). As can be seen in Figs 6(a) and 7(a), the crosses represent the positions of the UAV sensor at different position along the UAV fight trajectory and the square at the origin of the coordinate represents the position of the ship/land-based sensor.

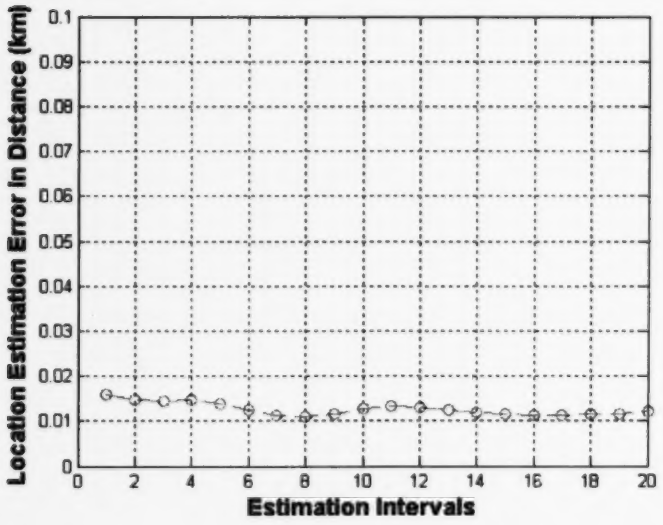
Based on the given geometries in Figs 6(a) and 7(a), the distance between sensor 1 and the emitter is about 14.3 km, while the distance from sensor 2 to the emitter varies from 3.3 km to 4.2 km. The associated range difference varies from 10.1 km to 11.0 km, corresponding to TDOA variation from 33.5 to 36.5 microseconds (μsec). The azimuth and elevation angles from sensor 1 to the emitter are about 0.785 radians (i.e., about 45 degrees) and 0.141 radians (i.e., about 8.1 degrees) respectively.

To evaluate the performance of the proposed method in handling measurement noise, the noisy measurements were produced by adding white Gaussian noise into the TDOA and AOA measurements. Figs. 6 and 7 show location estimation based on noisy TDOA and AOA measurements. The simulated standard deviation of the TDOA measurements was 10.0 and 30.0 nanoseconds (nsec) respectively, while the simulated standard deviation for the AOA measurements was fixed at 0.2 degrees. The location estimation shown in Figs. 6 and 7 were performed by collecting six TDOA and one AOA measurements for each estimation interval. The associated estimation errors at every estimation interval are given in Figs. 6(b) and 7(b). To compare the simulation results at different TDOA noise levels, the estimation errors given in Figs. 6(b) and 7(a) were re-plotted and given in Fig. 8. As can be seen, an increase in TDOA noise level increases estimation error, as we would expect.

A numerical simulation with increased number of TDOA measurements was also performed and the associated estimation results are given in Fig. 9. As can be seen, an increase in the number of TDOA measurements improves estimation accuracy.

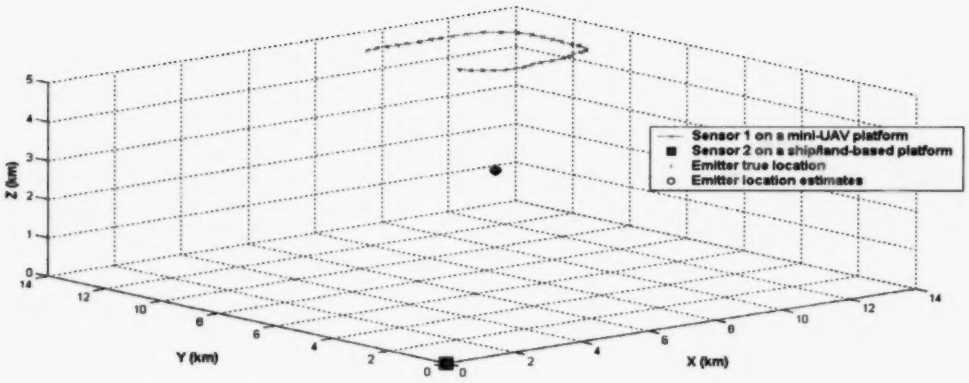


(a)

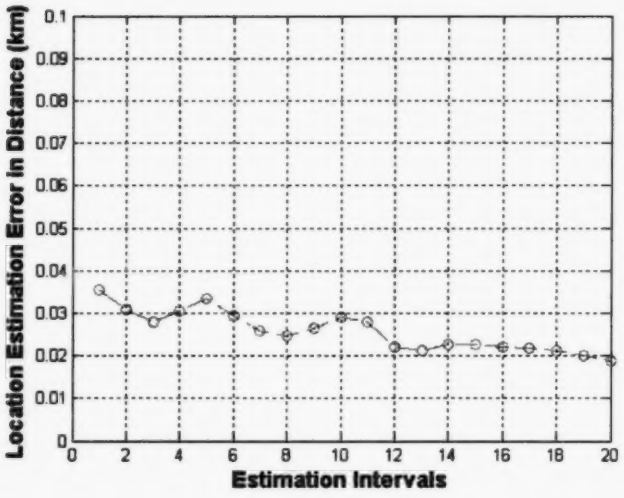


(b)

Figure 6. *. Monte-Carlo (10,000 runs) location estimation with TDOA's std dev=10.0 nsec and AOA's std dev=0.2 deg. (a) Location estimates against emitter true position; (b) Location estimation error.*



(a)



(b)

Figure 7. *Monte-Carlo (10,000 runs) location estimation with TDOA's std dev=30.0 nsec and AOA's std dev=0.2 deg. (a) Location estimates against emitter true position; (b) Location estimation error.*

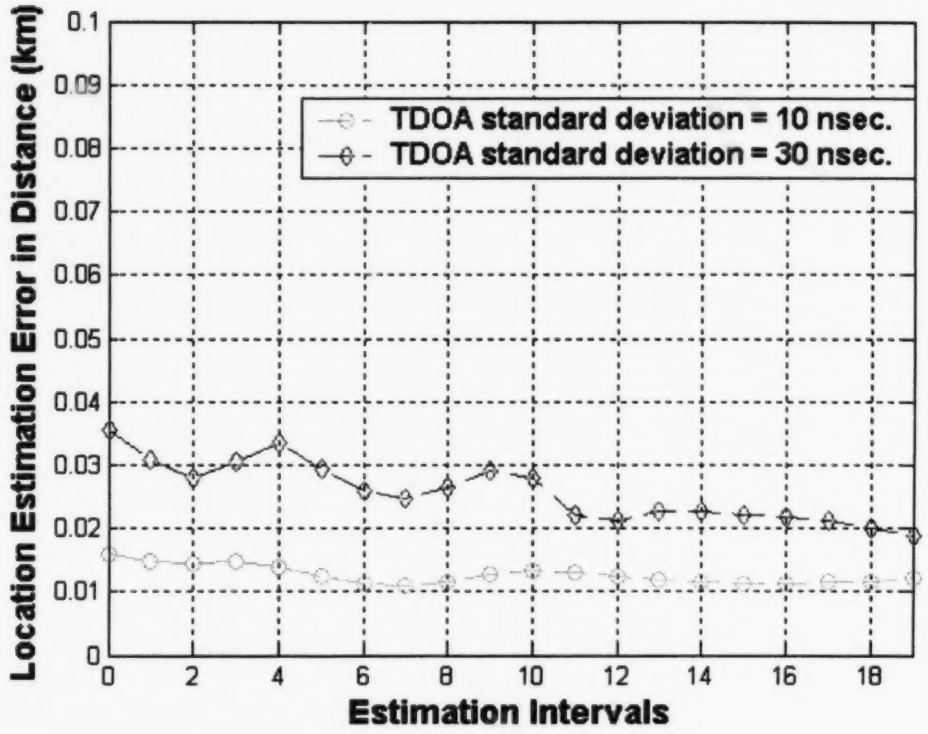


Figure 8. *Monte-Carlo (10,000 runs) location estimation errors with TDOA's std dev=10.0 and 30.0 nsec*

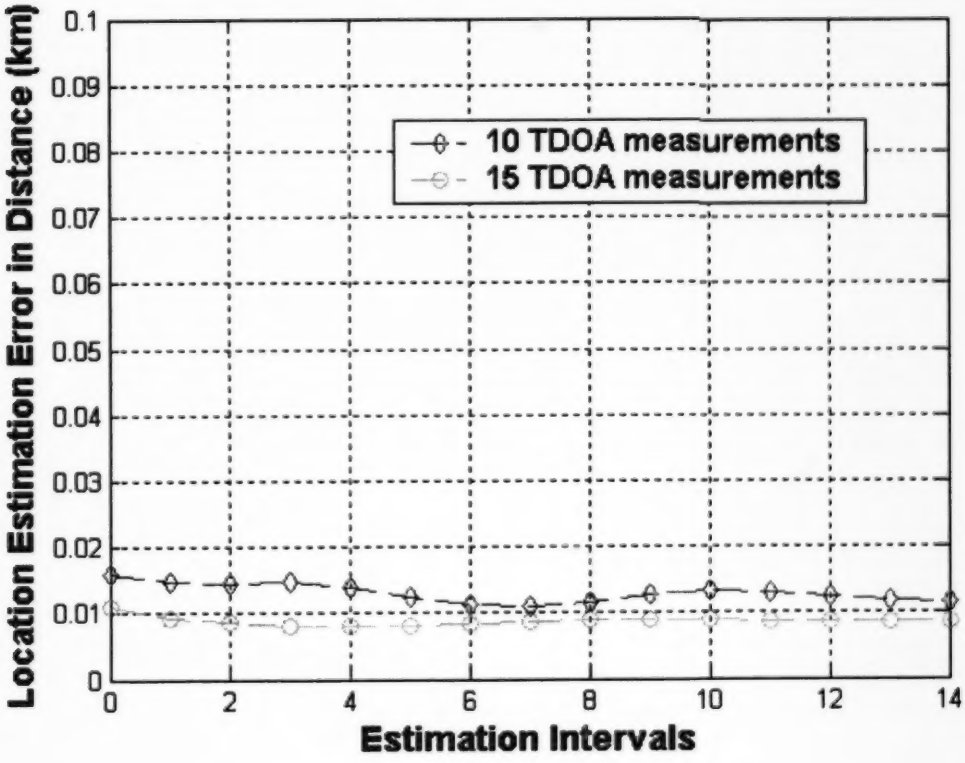


Figure 9. *Monte-Carlo (10,000 runs) location estimation errors with different number of TDOA measurements*

5. Summary and Conclusions

This report proposes the development of a multi-platform location technique using a hybrid TDOA/AOA scheme for passive radar emitter geolocation applications. Implicit in the development are the assumptions that the mechanisms for measuring TDOAs and AOAs on radar pulses exist, and that TDOA and AOA measurement errors are small. By incorporating a set of the AOA equations, we extend the previous TDOA only location estimators to solve the hybrid TDOA/AOA equations for 3-D location applications. By introducing an intermediate variable r_1, the nonlinear equations related to position estimates are transformed into a set of the linear equations which are a function of the unknown variables and the intermediate variable. Monte-Carlo simulation results demonstrate that the proposed location scheme is effective and that it converges fast with no need of an initial guess of emitter position. However, further studies need to be done in the future to investigate the performance of the proposed scheme in handling higher TDOA and AOA noise. Future studies also include applying another LS computation by applying the computed result $(\hat{x}_e, \hat{y}_e, \hat{z}_e, \hat{r}_1)$ to the known relationship to obtain an improved position estimate $(\hat{x}_e, \hat{y}_e, \hat{z}_e)$ as well as incorporating TDOA/AOA measurement variances as a weighting matrix in the LS to achieve a maximum-likelihood (ML) position estimate.

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The problem of geolocation of radar emitters has been studied extensively in the past decades. Several location methods such as the methods based on measuring frequency difference of arrival (FDOA), time difference of arrival (TDOA), angle of arrival (AOA), etc., have been investigated in the past. However, each method has its own advantages and limitations in terms of location accuracy. This report presents a multi-platform geolocation scheme by combining TDOA and AOA methods for three-dimensional (3-D) geolocation of radar emitters. A mathematical model is developed by combining sensor information such as AOA measurements, TDOA measurements and sensor position information from all platforms. A least-squares (LS) solution based on the model is derived by processing the sensor information. Through the use of the proposed hybrid TDOA/AOA location scheme, location ambiguity due to using TDOA method alone can be resolved and better location accuracy can be achieved. In addition, unlike single platform geolocation techniques, multi-platform geolocation generally results in quick localization of radar emitters. Therefore, it will reduce response time to incoming threats and improve situation awareness. A Monte-Carlo simulation is presented using additive white Gaussian noise to demonstrate the performance of the proposed hybrid TDOA/AOA geolocation method. (U)

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Multi-Platform, Geolocation, Time Difference Of Arrivel (TDOA), Angle Of Arrivel (AOA), Electronic Support (ES), Monte-Carlo simulation

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